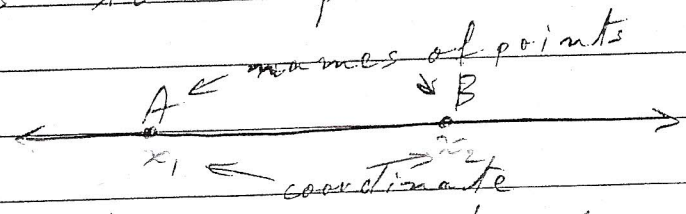


Postulate or Axiom: A rule that is accepted without proof

Theorem: A rule can be proved.

Postulate 1: Ruler Postulate

The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the coordinate of the point.



The distance between points A and B, written AB, is the difference of the coordinates of A & B

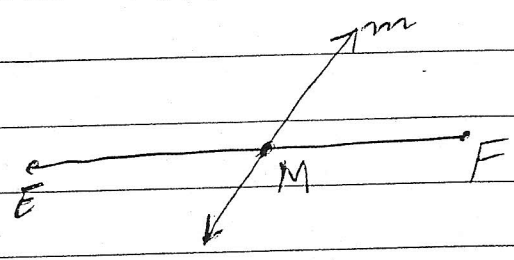
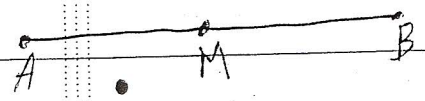
$$AB = |x_2 - x_1|$$

Congruent segments: are line segments that have same length.

Notations  $\overline{AB} \cong \overline{EF}$

Definitions: The midpoint of a segment is the point that divides the segment into 2 congruent segments.

A segment bisector is a point, ray, line, line segment, or plane that intersects the segment at its midpoint



# Geometry

Grade 7

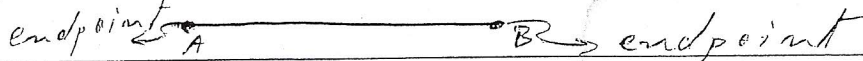
(P1)

Definition: Undefined terms are words do not have formal definitions, but there is agreement about they mean.

example: point, line, plane.

Definition: Defined terms can be described using known words such as point or line.

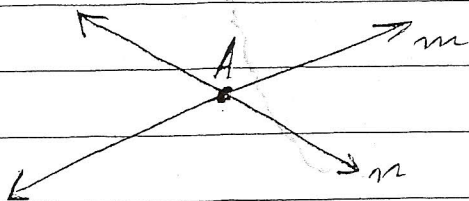
Segment: The line segment  $AB$  (written as  $\overline{AB}$ ) consists of the endpoints  $A$  and  $B$  and all points on  $\overleftrightarrow{AB}$  that are between  $A$  and  $B$ . Note that  $\overline{AB}$  can also be named  $\overline{BA}$ .



Ray: The ray  $AB$  (written as  $\overrightarrow{AB}$ ) consists of the endpoint  $A$  all points on  $\overleftrightarrow{AB}$  that lie on the same side of  $A$  as  $B$ .



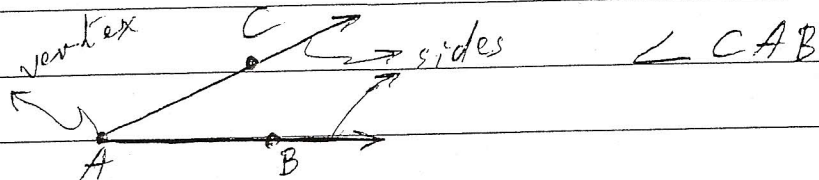
Intersection: The intersection of the figures is the set of points the figures has in common.



Distance formula:  $A(x_1, y_1)$   $B(x_2, y_2)$

$$(AB) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

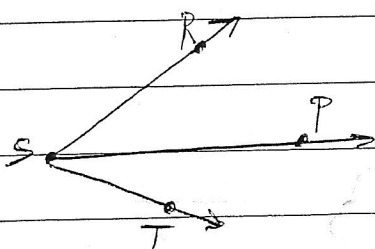
Definitions: An angle consists of 2 different rays with the same endpoint. The rays are the side of the angle. The endpoint is the vertex of the angle.



Postulate 4: Angle addition Postulate

If P is in the interior of  $\angle RST$ , then the measure of  $\angle RST$  is equal to the sum of the measures of  $\angle RSP$  and  $\angle PST$ .

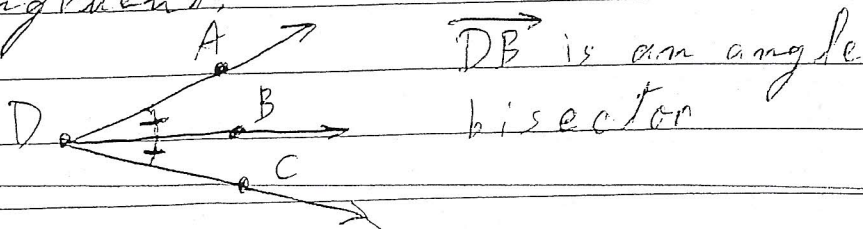
$$m\angle RST = m\angle RSP + m\angle PST$$



Definitions: Two angles are congruent angles if they have the same measure.

$$m\angle A = m\angle B, \angle A \cong \angle B$$

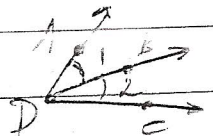
An angle bisector is a ray divides an angle in angles that are congruent.



Definitions: Two angles are complementary angles if the sum of their measures is  $90^\circ$ , each angle is the complement of the other.

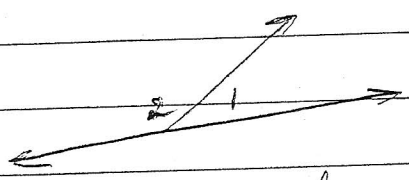
Two angles are supplementary angles if the sum of their measures is  $180^\circ$ . Each angle is the supplement of the other.

Adjacent angles are 2 angles that share a common vertex and side, but have no common interior point.

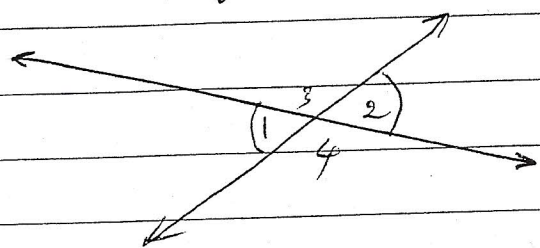


$\angle 1$  &  $\angle 2$  are adjacent

Two adjacent angles are a linear pair if their noncommon sides are opposite rays. The angles in a linear pair are supplementary angles.



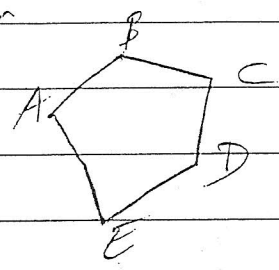
Two angles are vertical angles if their sides form two pairs of opposite rays.



$(\angle 3 \& \angle 4)$ , &  $(\angle 1 \& \angle 2)$  are vertical angles

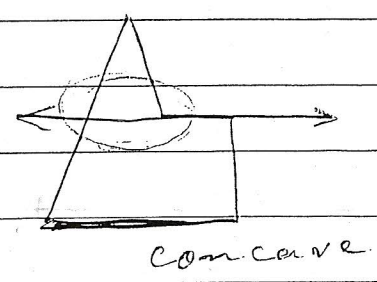
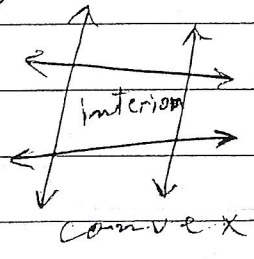
Identifying Polygons: In geometry, a figure that lies in a plane is called a plane figure. A polygon is a closed plane figure with the following properties.

1. It is formed by three or more line segments called sides.
2. Each side intersects exactly 2 sides, one at each endpoint, so that no 2 sides with common endpoint are collinear.



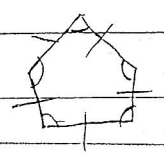
$\left. \begin{matrix} ABCDE \\ BCDEA \end{matrix} \right\} \text{notation}$

A polygon is convex if no line that contains a side of the polygon contains a point in the interior of the polygon. Otherwise is called concave.



The term  $n$ -gon, where  $n$  is the number of a polygon's sides. example: 12-gon with 12 sides.

A regular polygon is a convex polygon that is both equilateral and equiangular.



conjecture is an unproven statement that is based on observations. It must be true for all cases.

to use inductive reasoning when you find a pattern in specific cases and then write a conjecture for the general case.

A counterexample is a specific case for which the conjecture is false

Conditional statement: If (hypothesis) then (conclusion)

example: If it is raining then there are clouds in the sky

The negation of a statement is the opposite of the original statement.

ex: statement: the ball is red.  
negation: the ball is not red.

The converse of a conditional statement exchange the hypothesis and conclusion.

Inverse of a conditional statement, negate both the hypothesis and the conclusion

Contrapositive first write the converse and the nega. both the hypothesis and the conclusion.

ex: C.S.: If  $m\angle A = 99^\circ$  then  $\angle A$  is obtuse. (True)

Conv.: If  $\angle A$  is obtuse then  $m\angle A = 99^\circ$ . (False)

Inv.: If  $m\angle A \neq 99^\circ$  then  $\angle A$  is not obtuse. (False)

Contrapos.: If  $\angle A$  is not obtuse then  $m\angle A \neq 99^\circ$ . (True)

Definition: If 2 lines intersect to form a right angle, then they are perpendicular lines.

Biconditional statement is when conditional statement and its converse are both true.

If and only if

example: 2 lines are  $\perp$  iff they intersect to form a right angle.

Deductive reasoning uses facts, definition, accepted properties and laws of logic to form a logical argument.

Postulate 5: Through any 2 points there exists exactly one line.

Postulate 6: A line contains at least 2 points.

Postulate 7: If 2 lines intersect, then their intersection is exactly one point.

Postulate 8: Through any 3 noncollinear points there exists exactly one plane.

Postulate 9: A plane contains at least 3 noncollinear points.

Postulate 10: If 2 points lie in a plane, then the line containing them lies in the plane.

Postulate 11: If 2 planes intersect, then their intersection is a line.

Theorem 2.5: Congruent supplements theorem

If 2 angles are supplementary to the same angle then they are congruent.

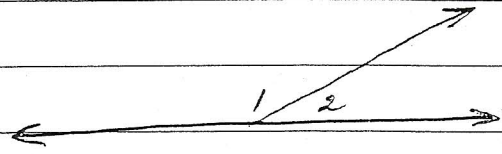
If  $\angle 4$  and  $\angle 5$  are supplementary and  $\angle 6$  and  $\angle 5$  are supplementary then  $\angle 4 \cong \angle 6$ .

Postulate 12: Linear pair postulate

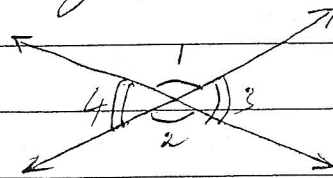
If 2 angles form a linear pair, then they are supplementary

$\angle 1$  &  $\angle 2$  form a linear pair

$\angle 1$  &  $\angle 2$  are supplementary



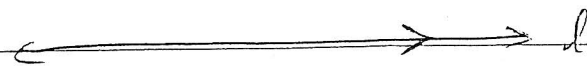
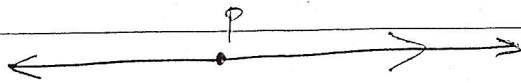
Theorem 2.6: Vertical angles congruence theorem  
vertical angles are congruent.



$\angle 1$  &  $\angle 2$  are vertical,

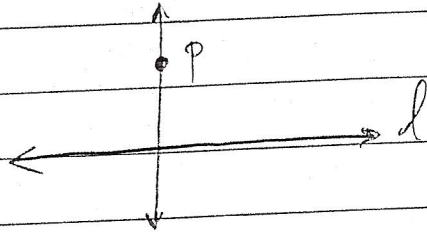
Postulate 13: Parallel Postulate

If there is a line and a point not on the line, there is exactly one line through the point parallel to the given line. There is exactly one line through P // to l



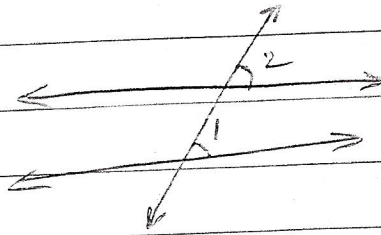
Postulate 14: Perpendicular Postulate

If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given point, there is exactly one line through  $P$  to  $l$ .

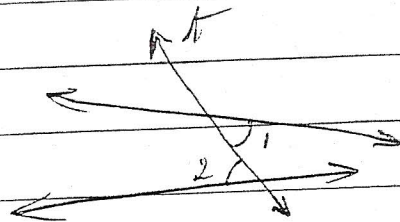


Angles formed by transversals

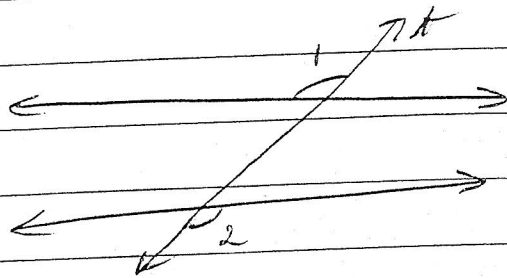
corresponding angles:  
 $\angle 1$  &  $\angle 2$



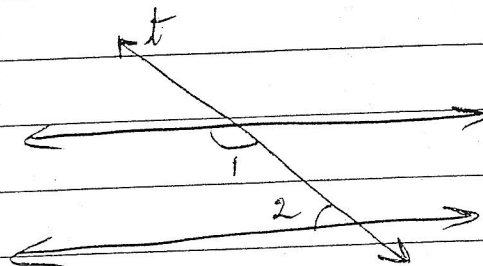
Alternate interior angles:  
 $\angle 1$  &  $\angle 2$



alternate exterior angles:  
 $\angle 1$  &  $\angle 2$

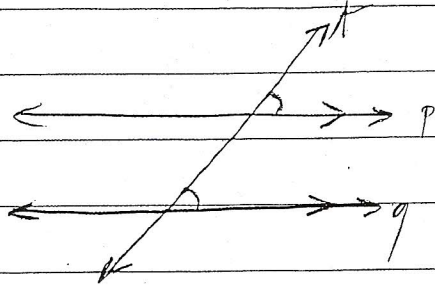


consecutive interior:  
 $\angle 1$  &  $\angle 2$



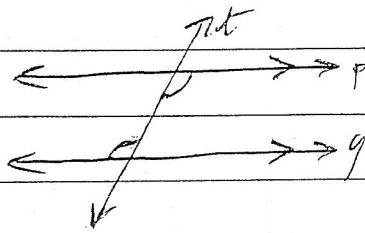
Postulate 15: Corresponding angles Postulate

If 2 // lines are cut by a transversal, then the pairs of corresponding angles are congruent.



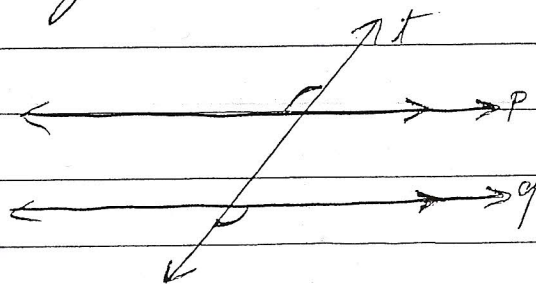
Theorem 3.1: Alternate interior angles theorem

If 2 // lines are cut by a transversal, then the pairs of alternate interior angles are  $\cong$ .



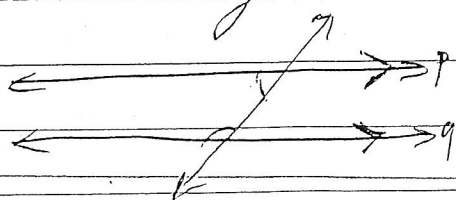
Theorem 3.2: Alternate exterior angles theorem

If 2 // lines are cut by a transversal, then the pairs of alternate exterior angles are  $\cong$ .



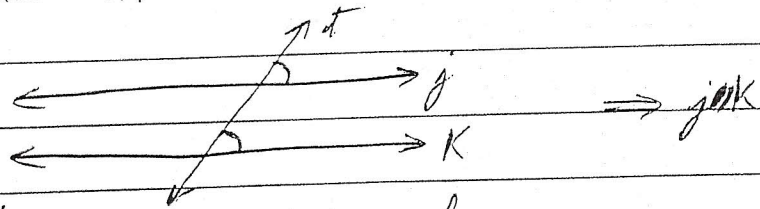
Theorem 3.3: Consecutive interior angles theorem

If 2 // lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.



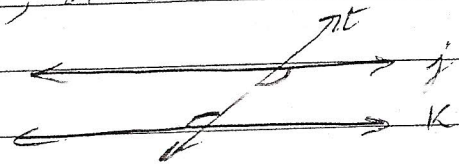
Postulate 16: corresponding angles converse

If 2 lines are cut by a transversal so the corresponding angles are  $\cong$ , then the lines are  $\parallel$ .



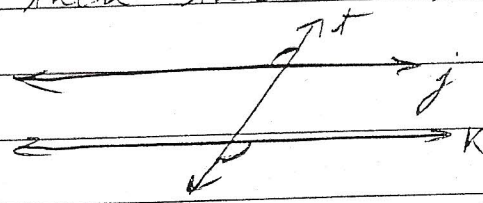
Theorem 3.4: Alternate interior angles converse

If 2 lines are cut by a transversal so the alternate interior angles are  $\cong$ , then the lines are  $\parallel$ .



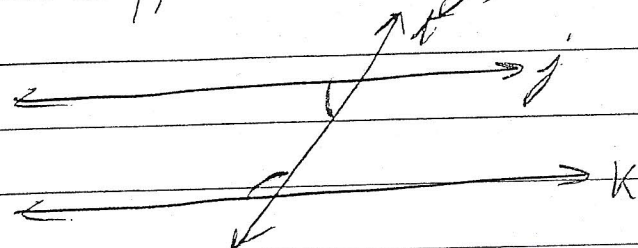
Theorem 3.5: Alternate exterior angles converse

If 2 lines are cut by a transversal so the alternate exterior angles are  $\cong$ , then the lines are  $\parallel$ .



Theorem 3.6: Consecutive interior angles converse

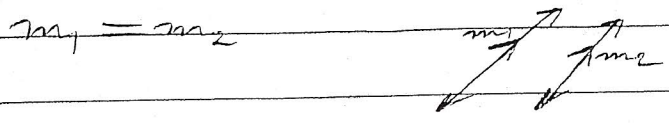
If 2 lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are  $\parallel$ .



Theorem 3.7: Transitive property of parallel lines

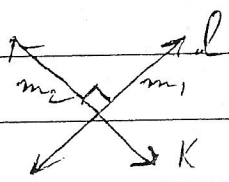
If 2 lines are  $\parallel$  to the same line, then they are  $\parallel$  to each other.

Postulate 17: Slopes of parallel lines  
In a coordinate plane, two nonvertical lines are  $\parallel$  iff they have the same slope.



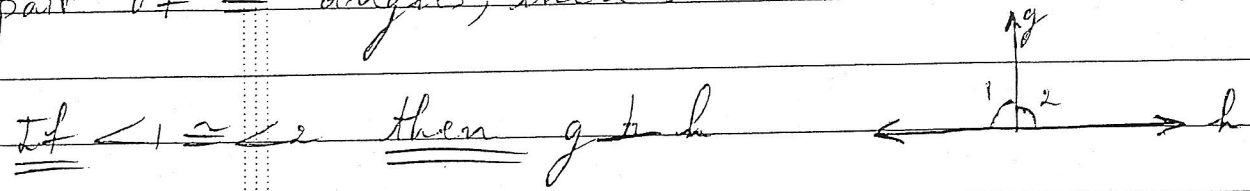
Postulate 18: slopes perpendicular lines

$m_1 \cdot m_2 = -1$



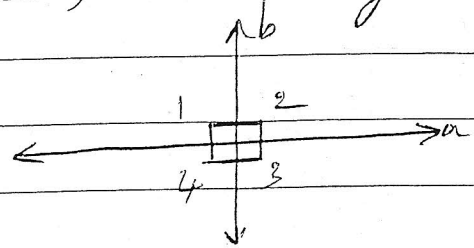
slope-intercept form of a linear equation;  $y = mx + b$   
standard form of linear equation;  $ax + by = c$

Theorem 3.8: If 2 lines intersect to form a linear pair of  $\cong$  angles, then the lines are  $\perp$ .



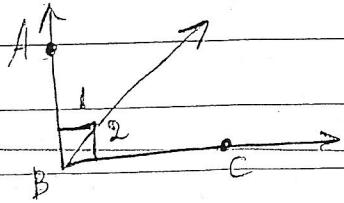
Theorem 3.9: If 2 lines are  $\perp$ , then they intersect to form 4 right angles.

If  $a \perp b$  then  $\angle 1, \angle 2, \angle 3$  and  $\angle 4$  are right angles.



Theorem 3.10: If 2 sides of 2 adjacent acute angles are  $\perp$ , then these angles are complementary.

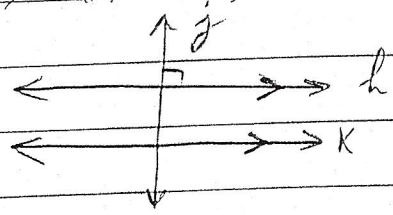
If  $\overrightarrow{BA} \perp \overrightarrow{BC}$  then  $\angle 1$  and  $\angle 2$  are complementary.



Theorem 3.11: Perpendicular transversal theorem

If a transversal is  $\perp$  to one of two  $\parallel$  lines, then it is  $\perp$  to the other.

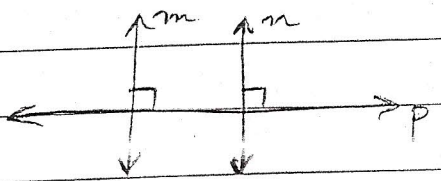
If  $h \parallel k$  and  $j \perp h$  then  $j \perp k$



Theorem 3.12: Lines perpendicular to a transversal theorem

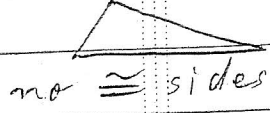
In a plane, if 2 lines are  $\perp$  to the same line, then they are  $\parallel$  to each other.

If  $m \perp p$  and  $n \perp p$  then  $m \parallel n$



Classifying triangles by sides:

scalene  $\Delta$



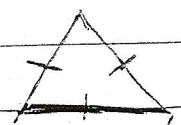
no  $\cong$  sides

isosceles  $\Delta$



at least 2  $\cong$  sides

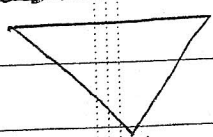
equilateral  $\Delta$



3  $\cong$  sides

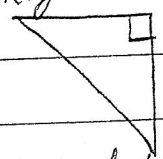
Classifying triangles by angles:

acute  $\Delta$



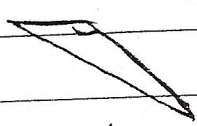
3 acute  $\angle$

Right  $\Delta$



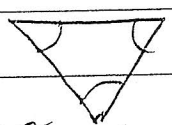
1 right  $\angle$

obtuse  $\Delta$



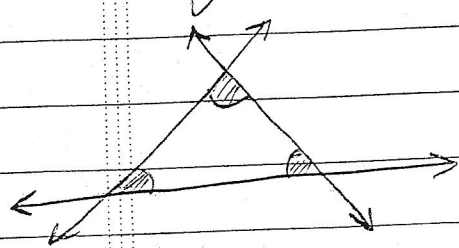
1 obtuse  $\angle$

equilateral  $\Delta$

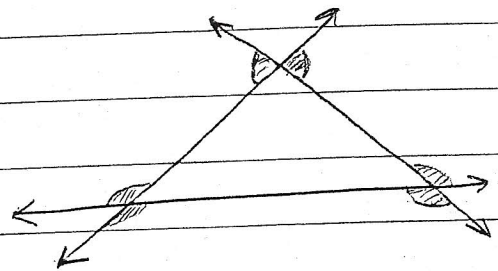


3  $\cong$   $\angle$

Interior angles



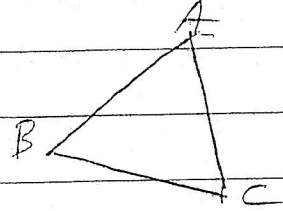
Exterior angles



Theorem 4.1: Triangle sum theorem

The sum of the measures of the interior  $\angle$ s of a triangle is  $180^\circ$

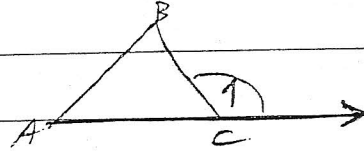
$$m\angle A + m\angle B + m\angle C = 180^\circ$$



Theorem 4.2: Exterior angle theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the 2 nonadjacent interior angles.

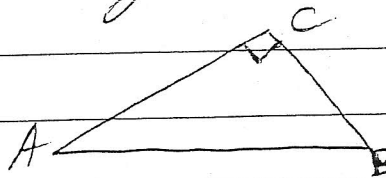
$$m\angle 1 = m\angle A + m\angle B$$



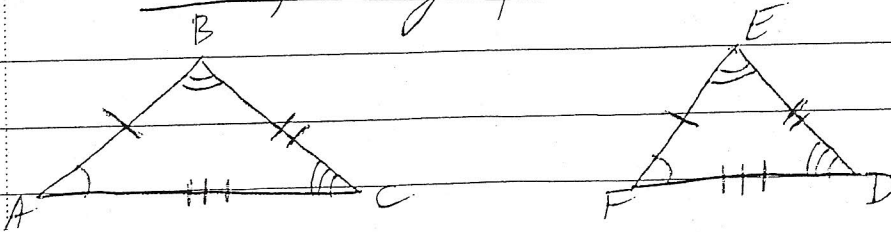
Corollary: Corollary to the triangle sum theorem

The acute  $\angle$ s of a right triangle are complementary

$$m\angle A + m\angle B = 90^\circ$$



In two congruent figures all the parts of one figure are  $\cong$  to the corresponding parts of the other figure.



Theorem 4.3: Third angles theorem

If 2  $\angle$ s of one triangle are  $\cong$  to 2  $\angle$ s of another triangle, then the third angles are also  $\cong$ .

Theorem 9.4: Properties of congruent triangles

reflexive:  $\triangle ABC \cong \triangle ABC$

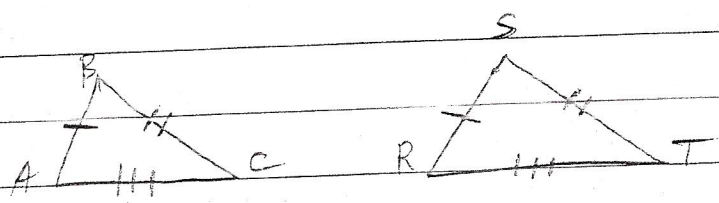
symmetric: If  $\triangle ABC \cong \triangle DEF$  then  $\triangle DEF \cong \triangle ABC$

transitive: If  $\triangle ABC \cong \triangle DEF$  and  $\triangle DEF \cong \triangle JKL$   
then  $\triangle ABC \cong \triangle JKL$

Postulate 19: (SSS) Side-Side-Side congruence postulate  
If 3 sides of one triangle are  $\cong$  to 3 sides of a second triangle, then the 2  $\triangle$  are  $\cong$ .

If  $\overline{AB} \cong \overline{RS}$  then  $\triangle ABC \cong \triangle RST$

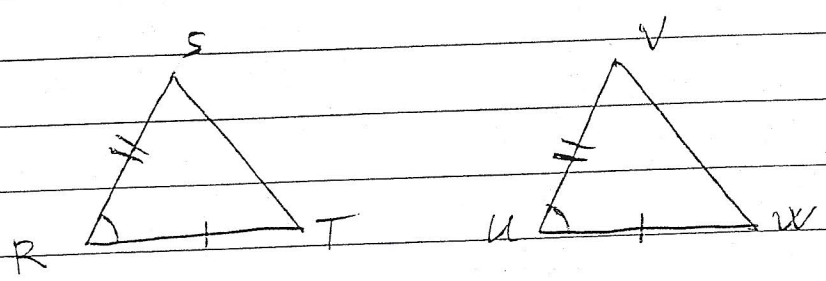
$\overline{BC} \cong \overline{ST}$   
 $\overline{CA} \cong \overline{TR}$



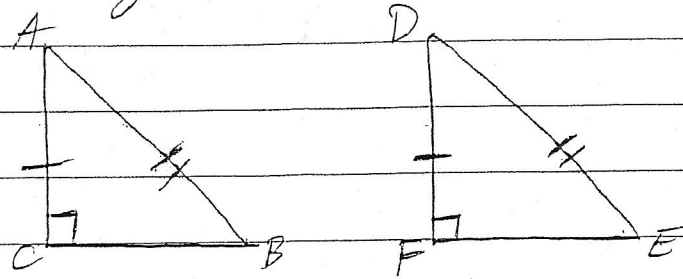
Postulate 20: Side-Angle-Side (SAS) congruence postulate  
If 2 sides and the included angle of one triangle are  $\cong$  to 2 sides and the included angle of a second triangle then the 2  $\triangle$  are  $\cong$ .

If  $\overline{RS} \cong \overline{UV}$  then  $\triangle RST \cong \triangle UVW$

$\angle R \cong \angle U$   
 $\overline{RT} \cong \overline{UW}$

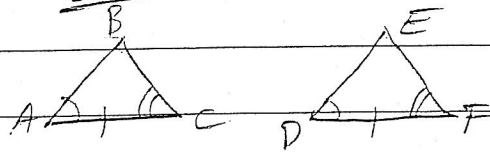


Theorem 4.5: Hypotenuse-leg (HL) congruence theorem  
 If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the 2  $\Delta$ s are  $\cong$ .



Postulate 2.1: Angle-Side-Angle (ASA) congruence postulate  
 If 2  $\angle$ s and included side of one triangle are  $\cong$  to 2  $\angle$ s and included side of a second triangle, then the 2  $\Delta$ s are  $\cong$ .

If  $\angle A \cong \angle D$ ,  $\overline{AC} \cong \overline{DF}$  then  $\Delta ABC \cong \Delta DEF$   
 $\angle C \cong \angle F$



Theorem 4.6: Angle-Angle-Side (AAS) congruence theorem  
 If 2  $\angle$ s and a non-included side of one triangle are congruent to 2  $\angle$ s and the corresponding non-included side of a second triangle, then the 2  $\Delta$ s are  $\cong$ .

If  $\angle A \cong \angle D$ ,  $\angle C \cong \angle F$  then  $\Delta ABC \cong \Delta DEF$   
 $\overline{BC} \cong \overline{EF}$

